Penman Equation

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Introduction

The Penman equation is essentially an energy balance equation, which equates all incoming and outgoing energy fluxes at earth's surface. It was developed by Howard Penman in 1948 and since then we have come across various variations of the Penman equation are used to estimate evaporation from water, and land. Since evaporation process depends on solar radiation, air temperature, relative humidity (i.e., vapor pressure deficit) and wind speed, Penman's equation uses them as parameters to give a prediction for the value of Evaporation (E). The equation reads:

$$R_n = G + H + \lambda E$$

where R_n is the energy flux density of net incoming radiation, G is the heat flux density into the water body, H and λE is the flux density of sensible and latent heat into the air respectively.
If we have sufficient knowledge and techniques for measuring R and G, then E can be calculated by the ratio $H/\lambda E$ (Bowen ratio) is known. This ratio can be derived from the transport equations of heat and water vapour in air. By applying the concept of similarity of transport of heat and water vapour, the Bowen ration can be rewritten as

$$\frac{H}{\lambda E} = \frac{c_1(T_s - T_a)}{c_2(e_s - e_d)}$$

where c_1/c_2 is psychrometric constant γ in $kPa/^\circ C$, T_s and T_a are the temperature at the evaporating surface and at a certain height above the surface, e_s and e_d are the saturated vapour pressure at evaporating surface and vapour pressure at the height where T_a is measured.
Now, as it is generally difficult to measure the surface temperature, T_s, so the value $T_s - T_a$ is calculated using slope of saturated vapour pressure curve and it is equal to

$$(e_s - e_a)\frac{dT_a}{de_a}$$

Finally we introduce a term called isothermal evaporation, λE_a, which we get from above relations under isothermal conditions, and further division and rearrangement we get formula for Penman (1948),

$$E_o = \frac{\Delta(R_n - G)/\lambda + \gamma E_a}{\Delta + \gamma}$$

where E_o is open water evaporation rate in $(kg/m^2 s)$, Δ is de_a/dT_a in $(kPa/^\circ C)$, R is the net radiation in (W/m^2), λ is latent heat of vaporization in (J/Kg), γ is psychrometric constant $(kPa/^\circ C)$ and E_a is isothermal evaporation rate $kg/m^2 s$.

Calculation Steps

1. Mean temperature
   We use the average of maximum and minimum temperatures in $°C$ to get our mean temperature. Direct calculations using only the available mean temperature data is avoided due to the non-linearity of the saturation vapor pressure - temperature relationship (Allen et al., 1998).

$$T_{mean} = \frac{T_{max} + T_{min}}{2}$$

2. Mean solar radiation (R_s)
   The average daily net radiation expressed in ($MJm^{-2}day^{-1}$) is required.

$$R_{s(MJm^{-2}day^{-1})} = R_{s(Wm^{-2}day-1)} \times 0.0864$$

3. Wind speed (u_2)
   The average daily wind speed in meters per second (ms^{-1}) measured at 2 m above the ground level is required. The wind speed measured at heights other than 2 m can be adjusted according to the follow equation:

$$u_2 = u_h \frac{4.87}{ln(67.8h - 5.42)}$$

4. Slope of saturation vapor pressure curve (Δ)
   The slope of the saturation vapor pressure vs temperature graph is calculated by:

$$\Delta = \frac{4098[0.6108 \times exp(\frac{17.27 \times T_{mean}}{T_{mean}+237.3})]}{(T_{mean} + 237.3)^2}$$

5. Atmospheric Pressure (P)
   The atmospheric pressure P, at height h is the pressure exerted by the weight of the earthâs atmosphere.

$$P = 101.3[\frac{293 - 0.0065h}{293}]^{5.26}$$

6. Psychrometric constant (γ)
   The psychrometric constant relates the partial pressure of water in air to the air temperature so that vapor pressure can be estimated using paired dry and wet thermometer bulb temperature readings. Another way to describe the psychrometric constant is the ratio of specific heat of moist air at constant pressure (C_p) to latent heat of vaporization.

$$\gamma = \frac{C_p P}{\epsilon \lambda} = 0.000665P$$

7. Delta Term (DT)
   The delta term is used to calculate the "Radiation Term" of the overall E_o equation

$$DT = \frac{\Delta}{\Delta + \gamma(1 + 0.34u_2)}$$

8. Psi Term (PT)
   The psi term is used to calculate the "Wind Term" of the overall E_o equation

$$PT = \frac{\gamma}{\Delta + \gamma(1 + 0.34u_2)}$$

9. Temperature Term (TT)
   The temperature term is used to calculate the "Wind Term" of the overall E_o equation

$$TT = [\frac{900}{T_{mean} + 273}] \times u_2$$

10. Mean saturation vapor pressure (e_s)
    As saturation vapor pressure is related to air temperature, it can be calculated from the air temperature.

$$e_{(T)} = 0.6108 \times exp[\frac{17.27T}{T + 237.3}]$$

The mean saturation vapor pressure for should be computed as the mean between the saturation vapor pressure maximum and minimum air temperatures for that period.

$$e_s = \frac{e_{(T_{max})} + e_{(T_{min})}}{2}$$

11. Actual vapor pressure (e_a)
    The actual vapor pressure can also be calculated from the relative humidity. Depending on the availability of the humidity data, different equations should be used.

$$e_a = \frac{e_{(T_{min})}[\frac{RH_{max}}{100}] + e_{(T_{max})}[\frac{RH_{min}}{100}]}{2}$$

For missing or questionable quality of humidity data, the e_a can be obtained by assuming when the air temperature is close to T_{min}, the air is nearly saturated with water vapor and the relative humidity is near 100%, in other words, T_{dew} is near the T_{min}.

$$e_a = e_{(T_{min})} = 0.6108exp[\frac{17.27T_{min}}{T_{min} + 237.3}]$$

12. Total E_o Calculation

    (a) Radiation term (E_{rad})

$$E_{rad} = 0.408 \times DT \times R_s$$

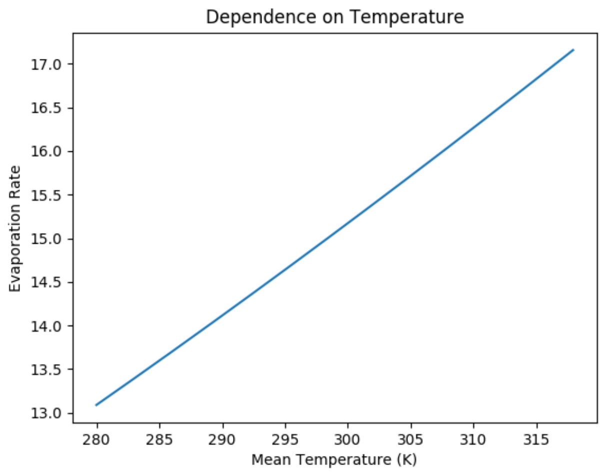
    (b) Wind term (E_{wind})

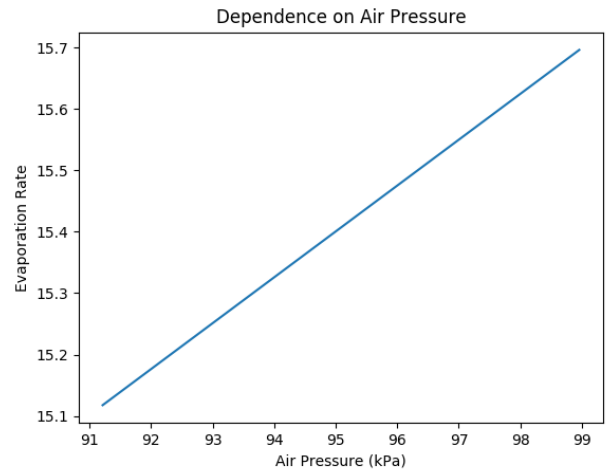
$$E_{wind} = PT \times TT \times (e_s - e_a)$$

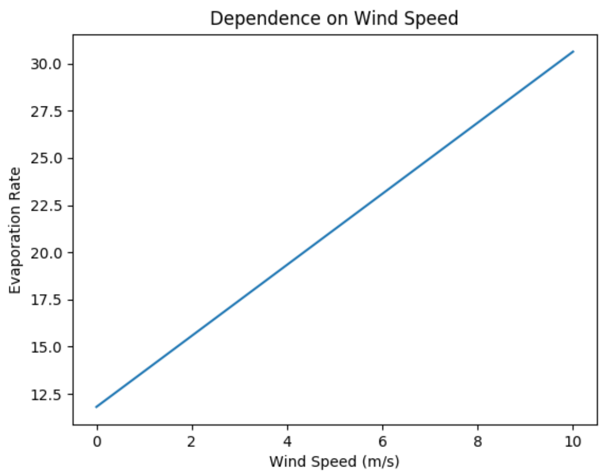
Finally:

$$E_o = E_{rad} + E_{wind}$$

13. Plotting (Scales needed to be fixed): Default Average Temperature: 302 K, Default Wind speed: 4 m/s, Default Height: 550 m above the sea level. Summary







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